1 Introduction

This paper deals with the interplay between phase particles and quantification in German. We shall give a semantic analysis of phase structures denoted by sentences such as (1) in which one of the phase particles in (2a) is applied to a proposition whose subject is one of the quantifiers in (2b):

(1) (a) \textit{Einige sind nicht mehr da.}
Some people are not there anymore.

(b) \textit{Alle sind schon da.}
Everyone is already there.

(2) (a) \textit{schon} (already)

\begin{tabular}{ll}
  noch (still) & \textit{einige} (some) \\
  noch nicht (not yet) & \textit{nicht alle} (not all) \\
  nicht mehr (no longer) & \textit{niemand} (nobody)
\end{tabular}

The plan of the paper is as follows. In the second section we will set up a suitable formal framework by formalizing the basic meaning of phase particles as operating on predicates of times. In the third section we will give a detailed analysis of the scope relations between phase particles and quantifiers. This analysis turns out to require a somewhat intricate procedure of extracting negations from phase particles and quantifiers. The scope analysis will allow us to divide the 32 combinations consisting of one phase particle and one quantifier into eight equivalence classes each of which contains four combinations.

In the fourth section the formalization developed so far is shown to fail to account for an important feature of the intuitive meaning of the phase structures under consideration. In the fifth section I propose a solution

* I am indebted to Regine Eckardt, Markus Egg, Cornelia Endris, Tatjana Heyde-Zybatow and Ingolf Max for valuable remarks on this paper. Especially I would like to thank Christopher Piñón whose thorough comments on an earlier draft prevented me from saying a lot of unconvincing things, and who suggested to me to use generalized quantifiers in this paper. All errors and obscurities are of course only the author's responsibility.
to this problem within a generalized quantifier framework, giving up the assumption that phase particles operate on predicates of times.

2 Formalizing phase particles

The basic meaning of phase particles is a temporal one. It is confined to sentences in which phase particles take wide scope over an ‘imperfective’ (atelic) time-relative proposition such as \textsc{peter-sleep}(t). In the basic meaning, that is, phase particles are applied to predicates of times without focussing on the constituents of these predicates (Löbner 1989, 1990).

(3) Peter is already asleep.

We will assume that the basic meaning of phase particles consists of an assertion and a presupposition. A sentence such as (3) asserts that Peter is asleep at the speech time, and presupposes that he was not asleep at some time in the recent past. More generally, the assertion of \textit{already} states that the argument proposition \(P(t)\) holds at the time of assertion, that is, at the reference time \(t_r\) (which is identical to the speech time in the present tense). The presupposition of \textit{already} states that \(P(t)\) did not hold at some time before \(t_r\), giving rise to a phase structure consisting of a negative and an ensuing positive phase of \(P(t)\). During the negative phase, \(P(t)\) is false, and during the positive one, \(P(t)\) is true, the reference time \(t_r\) being part of the positive phase. \textit{Not yet}, on the other hand, presupposes (the expectation of\(^1\)) a change from a negative phase to a positive one as well, but asserts that the reference time \(t_r\) is part of the negative phase.

By analogy, \textit{still} asserts that the argument proposition is true at the reference time, but presupposes (the expectation of) a change from positive to negative polarity. That is, \textit{still} presupposes (the expectation) that a negative phase of \(P(t)\) will follow the positive one. Again, \textit{no longer} shares this presupposition, but asserts that the reference time belongs to the negative phase. Thus the phase structures triggered by the four phase particles can be illustrated by the following diagram:

---

\(^1\) It is a moot point whether the phase structure after the reference time \(t_r\) is to be regarded as truth-conditionally presupposed or rather as expected or implicated (see Doherty 1973, p. 155; König 1977, pp. 192f; Löbner 1989, p. 176; 1990, p. 118; 1999, p. 60; van der Auwera 1998, pp. 39f; Smessaert and ter Meulen 2004, p. 237). I shall not discuss this issue in this paper. For the sake of simplicity, I shall neglect epistemic and pragmatic differences between the phase structure before and after the reference time, giving a logical account of the symmetric full-blown phase structures displayed in the diagram in (4) below.
In formalizing the basic meaning of the phase particles we will use a
two-sorted first-order language containing variables \( t, t_1, t_2, \ldots \) of type \( \langle i \rangle \) standing for time points, and variables \( z, j, m, \ldots \) of type \( \langle e \rangle \) standing for individuals such as John and Mary. Moreover, we will need a binary relation of temporal precedence \( t < t_1 \) (\( t \) is before \( t_1 \)) obtaining between
time points.

Given this relation, the change from negative to positive polarity pre-
supposed by \textit{already} can be described by stating the existence of a final
time point \( t^* \) of the negative phase, such that for all times \( t \), the argument
proposition \( P(t) \) holds if and only if \( t \) is after \( t^* \):

\[ \exists t^* \forall t (t^* < t \leftrightarrow P(t)) \]

\textit{Not yet} is similar to \textit{already} in that it presupposes a change from neg-
ative to positive polarity, but differs from it in that the reference time \( t_r \) is part of the negative phase. The two phase particles share the same presupposition \((5)\), but differ from each other in asserting \( \neg P(t_r) \) or \( P(t_r) \) respectively. \textit{Still} and \textit{no longer}, on the other hand, presuppose a change from positive to negative polarity. Such phase structures can be expressed by stating that there is a final time point \( t^* \) of the positive phase such that \( P(t) \) is false if and only if \( t \) is after \( t^* \):

\[ \exists t^* \forall t (t^* < t \leftrightarrow \neg P(t)) \]

Of course, the phase structure triggered by the phase particles holds only for a contextually relevant time interval. This interval may cover a number of years like in \((7a)\), or only a few seconds like in \((7b)\).

\[ (7) \quad \begin{align*} 
(a) \quad & \text{Peter is still a child.} \\
(b) \quad & \text{The traffic light is already green.}
\end{align*} \]

\(^2\) For a similar analysis of the presupposition of \textit{already}, see Krifka (1995, p. 242).
Formally, the fact that phase particles are related to a contextually relevant time interval could be captured by restricting both quantifiers in (5) and (6) to a certain time interval. In this case, we would have to add the requirement that the final time $t_*$ of the negative or positive phase be neither an initial nor a final point of that interval, as otherwise we would obtain trivial phase structures lacking any change of polarity. For the sake of simplicity, however, we will not explicitly represent the contextually relevant interval in our formalization of the phase particles. Instead, we will tacitly assume that the domain $(i)$ of time points is already restricted to a contextually relevant time interval, and that there are always time points before and after $t_*$.

In order to describe the presuppositional structure of the phase particles, we will use a two-dimensional framework. That is, the phase particles will be represented as ordered pairs of classical first-order expressions:

\begin{equation}
\begin{array}{ll}
\text{assertion} & \text{presupposition} \\
\end{array}
\end{equation}

The relation between assertion and presupposition in such two-dimensional formulae may be thought of as the relation of standard classical conjunction, displaying non-standard behavior only under negation. Thus, $\lambda$-prefixes are allowed to be attached to two-dimensional formulae, the rules governing these prefixes being the same as those governing prefixes of classical conjunctions. Given this framework, the basic meaning of the phase particles can be described as follows:

(9) Let $t_*$ be a variable of type $(i)$ and $P$ a predicate of type $(i, t)$:

\begin{align*}
\text{ALREADY} & \quad =_{df} \lambda P t_* \left[ \exists t, \forall (t_* < t \rightarrow P(t)) \right] \\
\text{NOT-YET} & \quad =_{df} \lambda P t_* \left[ \exists t, \forall (t_* < t \rightarrow \neg P(t)) \right] \\
\text{STILL} & \quad =_{df} \lambda P t_* \left[ \exists t, \forall (t_* < t \rightarrow \neg \neg P(t)) \right]
\end{align*}

3 Suggestions of such two-dimensional frameworks include Karttunen and Peters (1979), Bergmann (1981).

4 Semantically, such two-dimensional formulae are evaluated relative to standard classical models: the formula $\left[ \begin{array}{c} A \\ B \end{array} \right]$ is true in a classical model if both $A$ and $B$ are true in it; it is false if $A$ is false and $B$ is true; otherwise it is incorrect.
We want the default negation of two-dimensional formulae to negate only the assertion and to preserve the presupposition. Moreover, we want this negation to apply also to λ-prefixed formulae. This leads to the following definition of a default negation $\neg_\lambda$, with $\lambda X$ being a possibly empty string of λ-abstractions:

$$\neg_\lambda \lambda X. \left[ \begin{array}{c} A \\ B \end{array} \right] = \lambda X. \left[ \begin{array}{c} \neg A \\ B \end{array} \right]$$

Finally, an equivalence relation $\equiv$ obtaining between (possibly λ-prefixing) two-dimensional formulae can be defined as follows:

$$A \equiv B \iff A_1 B_1 \equiv A_2 B_2 \iff A_1 \equiv A_2 \land B_1 \equiv B_2.$$
phase particle precedes the quantifier, e.g., *schon einige, noch einige*. These combinations often contain the expletive subject *es* (it):

(16) *Es sind schon einige da.*
There are already some people there.

(17) *Es sind noch einige da.*
There are still some people there.

In the remaining sixteen combinations, the quantifier precedes the phase particle, e.g., *einige schon, einige noch*:

(18) *Einige sind schon da.*
Some people are already there.

(19) *Einige sind noch da.*
Some people are still there.

Throughout this paper, I confine myself to considering the default word order in neutral sentences, neglecting leftward movements caused by information structure such as in the following example:

(20) *Alle sind noch Nicht da, aber einige.*
Not everyone is there yet, but some of them.

If we assume that there is a close connection between default word order and semantic scope, the sixteen PQ combinations in which the phase particle precedes the quantifier are – at first glance – unproblematic. It does not seem to be relevant that the subject of the proposition to which the phase particle is applied is a quantifier. The phase particle is applied to such a proposition in the very same way as to any other proposition. Applying already to *Einige sind da*, that is, to \( \lambda t. \exists z R(z, t) \), yields the following formula:

\[
(21) \quad \text{already}(\lambda t. \exists z R(z, t)) = \lambda t_r. \left[ \exists z R(z, t_r) \right] \supset \left[ \exists t, \forall t (t_r < t \leftrightarrow \exists z R(z, t)) \right]
\]

This seems to be a correct paraphrase of the meaning of (16). (21) predicts that during the presupposed negative phase, \( \lambda t. \exists z R(z, t) \) is false, i.e., no people are there, and that during the positive phase after \( t_* \), \( \lambda t. \exists z R(z, t) \) is true, i.e., some people are there. The reference time \( t_r \) is part of the positive phase, as \( \exists z R(z, t_r) \) is true. By analogy, (22) yields a correct paraphrase of (17), predicting that during the positive phase, which is located before \( t_* \) and which contains the reference time, \( \lambda t. \exists z R(z, t) \) is true, i.e., some people are there, and that no people will be there during the ensuing negative phase:

\[
(22) \quad \text{still}(\lambda t. \exists z R(z, t)) = \lambda t_r. \left[ \exists z R(z, t_r) \supset \exists t, \forall t (t_r < t \leftrightarrow \neg \exists z R(z, t)) \right]
\]
Thus, the two PQ combinations in (16) and (17) can be correctly analyzed by assuming that the phase particle takes wide scope over the quantifier, that is, that the default word order corresponds to semantic scope. However, the correspondence between word order and semantic scope breaks down when the quantifier precedes the phase particle. In (18) and (19) the quantifier *einige* obviously does not take wide scope over the phase particle, even though preceding it. Otherwise (18) would mean that there is at least one person, say John (*j*), who meets the phase structure of *schon*. That means that only John must be absent during the preceding negative phase presupposed by *schon* while all other people may be present:

\[
(23) \text{already}(\lambda t. R(j, t)) = \lambda t. \left[ R(j, t_r) \right] \exists t, \forall t (t < t_r \rightarrow R(j, t))
\]

Clearly, this is not what we mean by (18) (*Einige sind schon da*). If John is the only one still missing at a party, it would be rather odd to utter (18) after John has arrived, even though there is at least one person, namely John, who meets the phase structure of *schon*. Instead, (18) presupposes that during the preceding negative phase no people are present, just like the sentence (16) using the same phase particle and the same quantifier in the reversed PQ word order. Both sentences are equivalent to the wide-scope construction (24), and can, for our purposes, be taken to have exactly the same meaning.

\[
(24) \text{Es ist schon der Fall, dass einige da sind.}
\text{It is already the case that some people are there.}
\]

In the same way, (19) (*Einige sind noch da*) does not mean that only some people will not be present during the ensuing negative phase, but that no people will be present. Therefore, (19) has the same meaning as (17), and both sentences are correctly rendered by the wide-scope paraphrase (25):

\[
(25) \text{Es ist noch der Fall, dass einige da sind.}
\text{It is still the case that some people are there.}
\]

We can conclude that the word order of the phase particle and the quantifier does not matter in (16)-(19). In all of these examples, the phase particle takes scope over the quantifier, causing a mismatch between word order and scope in those cases when the quantifier precedes the phase particle ((18) and (19)).

However, the order of the phase particle and the quantifier does make a difference when the phase particle is non-factive, that is, when it contains a negation (*noch nicht, nicht mehr*). (26) is not equivalent to (27):

\[
(26) \text{Es ist noch der Fall, dass einige da sind.}
\text{It is still the case that some people are there.}
\]

\[
(27) \text{Es ist noch der Fall, dass einige da sind.}
\text{It is still the case that some people are there.}
\]
The first sentence states that no people are there at the reference time (=speech time), while the latter only states that some people are not there. Since indefinites such as *einige* usually cannot stand immediately after a negation, (26) sounds somewhat odd in the absence of *einmal*, but nevertheless the difference between (26) and (27) is perfectly understandable. The non-factive phase particle does not take scope over the quantifier in the QP sentence (27). The same is true for the following QP sentence containing a non-factive phase particle:

(28) *Einige sind nicht mehr da.*
Some people are not there anymore.

Neither (27) nor (28) can be correctly rendered by the wide-scope paraphrases (29) and (30), respectively. These paraphrases state that no people are present at the reference time (=speech time), whereas (27) and (28) only require that some people be absent at the reference time.

(29) *Es ist noch nicht der Fall, dass einige da sind.*
It is not yet the case that some people are there.

(30) *Es ist nicht mehr der Fall, dass einige da sind.*
It is not the case anymore that some people are there.

In view of the fact that the non-factive phase particle does not take wide scope over the quantifier in (27) and (28) one might conclude that the quantifier takes scope over the phase particle. In this case, however, both sentences would be true as soon as some people meet the phase structure of *noch nicht* or *nicht mehr* respectively, that is, if only some people are present during the presupposed positive phase. But clearly, (27) and (28) presuppose that all people are present during the (preceding or ensuing) positive phase. Hence the quantifier cannot take wide scope over the non-factive phase particle in these sentences.

In order to obtain a correct paraphrase of (27), we need to split up the phase particle *noch nicht* into a factive, purely positive part *noch* and an ‘internal’ negation *nicht*. The internal negation remains within the scope of the quantifier, whereas the factive part of the phase particle is given wide scope over the quantifier like in (18) and (19). This yields (31), which is a correct paraphrase of (27).

(31) *Es ist noch der Fall, dass einige nicht da sind.*
It is still the case that some people are not there.

In the case of (28) it may not be obvious how to split up *nicht mehr* (no longer) into a factive phase particle and an internal negation: *mehr* is an NPI substitute for *noch*\(^5\) while *nicht* is an external presupposition preserving negation of *noch* (see (13b)). From a logical point of view, however, *nicht mehr* is equivalent to *schon nicht* (already not) (see (15a)). This is confirmed by the fact that in many Slavic languages, the phase structure of *no longer* is expressed by *already not* (for instance, Czech *uz* *ne*). Thus *nicht mehr* can be split up into the factive phase particle *schon* and an internal negation. As in the case of (27), the correct paraphrase of (28) is obtained by giving wide scope to the factive phase particle and leaving the internal negation within the scope of the quantifier:

(32) *Es ist schon der Fall, dass einige nicht da sind.*
It is already the case that some people are not there.

The above way of splitting up phase particles can be symbolized by two functions \(\oplus\) and \(\ominus\) which yield the factive part of the phase particle \(P^\oplus\) and, if it exists, the internal negation \(P^\ominus\):

\[
\begin{array}{|c|c|c|c|}
\hline
P & P^\oplus & P^\ominus & \text{already} & \text{already not} \\
\hline
\emptyset & \emptyset & \emptyset & \emptyset & \emptyset \\
\hline
\end{array}
\]

Finally, the situation becomes even more complex when we consider the monotone decreasing quantifiers *niemand* and *nicht alle* in \(QP\)-sentences. In this case, the factive (part of the) phase particle does not take scope over the whole quantifier: (35) is not a correct paraphrase of (34) nor is (37) a correct paraphrase of (36).

(34) *Niemand ist noch da.*\(^6\)
Nobody is still there.

(35) *Es ist noch der Fall, dass niemand da ist.*
It is still the case that nobody is there.

(36) *Nicht alle sind nicht mehr da.*
Not everyone is no longer there.

---

\(5\) In certain contexts *mehr* and *noch* are interchangeable, for instance, after *kaum* (hardly). Both *kaum mehr* and *kaum noch* are acceptable. Compare also the interchangeability of Dutch *niets nog* (nothing still) and *niets meer* (nothing anymore) described by van der Auwera (1998, p. 101f).

\(6\) *Noch* is a positive polarity item (see p. 11 below). Therefore *Niemand ist noch da* sounds somewhat odd. One would prefer *Niemand ist mehr da* instead. For present purposes, we may neglect this difficulty, as the meaning of *Niemand ist noch da* is none the less perfectly understandable.
Es ist schon der Fall, dass nicht alle nicht da sind.
It is already the case that not everyone is not there.

It is helpful here to remind ourselves that every monotone decreasing quantifier can be thought of as the negation of a monotone increasing quantifier (Barwise and Cooper 1981, p. 186). For instance, niemand (nobody) is nicht einige (not some). The correct paraphrase of (34) and (36) is obtained by giving to the factive part of the phase particle scope over the monotone increasing part of the quantifier (alle and einige) while the negation of the quantifier is given wide scope over the whole sentence including the factive phase particle:

Es ist nicht der Fall, dass noch einige da sind.
It is not the case that still some people are there.

Es ist nicht der Fall, dass schon alle nicht da sind.
It is not the case that already all people are not there.

In Q P sentences, the external negation $Q^\ominus$ of the quantifier takes scope over sentences that contain a presupposition triggered by a phase particle. That is, within the two-dimensional framework introduced above, $Q^\ominus$ is applied to a two-dimensional formula. In this case, $Q^\ominus$ can be represented by the presupposition preserving default negation $\neg_a$ defined in (10). In P Q sentences, on the other hand, the negation $Q^\ominus$ does not take scope over the phase particle, but is applied to a one-dimensional formula. In this case, $\neg_a$ will be understood to be the standard classical negation $\neg$.

Given the procedure of splitting up phase particles and quantifiers specified in (33) and (40), the scope behavior of Q P sentences can be described by the following diagram:

| $Q^\ominus$ | $\emptyset$ | $\exists z$ | $\forall z$ | $\exists z$ |
| $Q^\ominus$ | $\emptyset$ | $\exists z$ | $\forall z$ | $\exists z$ |

The mismatch between word order and scope is confined to the ‘positive’ parts $P^\oplus$ and $Q^\oplus$ of the phase particle and the quantifier. These have to be interchanged in order to get the correct scope relations. The internal negation of the phase particle $P^\ominus$ and the external negation of the quantifier $Q^\ominus$ are not affected by this inversion. In P Q sentences, on the other hand, the order of

---

7 For a similar phenomenon, see the ‘split readings’ of quantifiers in de Swart (2000).
8 That is, $\neg_a$ is understood to be applicable to both one- and two-dimensional formulae. When being applied to one-dimensional formulae, $\neg_a$ reduces to classical negation.
the ‘positive’ and ‘negative’ parts of the phase particles and quantifiers remains unchanged and corresponds to the word order:

\[(42) \ P, Q \rightarrow P^\oplus, P^\ominus, Q^\oplus, Q^\ominus\]

As an example, the scope analysis specified in (41) and (42) will be carried out for the combinations *niemand noch nicht* and *schon alle*, which happen to yield equivalent phase structures:

\[(43) \ niemand\ noch\ nicht \leftrightarrow niemand^\ominus\ noch\ nicht^\oplus\niemand^\oplus\ noch\ nicht^\ominus = \neg_a\ still\ \exists z\ \neg\]

This means:

*niemand noch nicht* \(\leftrightarrow\ \neg_a\ \text{Still}\ \exists z\ \neg\)

When applying this scope analysis to all the 32 combinations consisting of a phase particle and a quantifier, it turns out that there are eight equivalence classes.\(^9\) This means that German is able to express only eight different phase structures by means of the four quantifiers and the four phase particles considered in this paper. Each of the equivalence classes contains four combinations. These combinations are listed in the first row of the table on p. 13f. The second row gives the formal representation obtained by the scope analysis specified in (41) and (42). The third row illustrates the intuitive meaning of the given equivalence class by a diagram.

Those combinations which cannot be uttered felicitously in neutral contexts are marked by ″. Within the equivalence classes 1-4, the combinations marked by ″ are inappropriate because they contain two negations while there are two equivalent combinations which do not contain any negation. Thus, it would violate pragmatic principles to use the doubly negated combinations in neutral contexts. Within the equivalence classes 5-8, however, both correct and incorrect combinations contain only one negation. Hence there must be other reasons for the incorrectness in these equivalence classes than the pragmatic constraints explaining the incorrectness in the equivalence classes 1-4. First, *schon* and *noch* are

\(^9\) That is, equivalence classes in the sense of the equivalence relation \(\leftrightarrow\) defined in (11).
positive polarity items\textsuperscript{10} (PPI) which usually cannot stand within the scope of monotone decreasing quantifiers ("\textit{niemand noch}, "\textit{nie nicht alle noch}, "\textit{niemand schon}, "\textit{nie nicht alle schon}). Second, \textit{eine} shows PPI properties as well\textsuperscript{11} ("\textit{nicht mehr einige}, "\textit{noch einige}"). Moreover, \textit{alle} normally does not allow for negations within its scope\textsuperscript{12} ("\textit{alle nicht mehr}, "\textit{alles noch nicht}). The same is true for \textit{schon}, which does not allow for negations within its scope either ("\textit{schon niemand}, "\textit{schon nicht alle}). Finally, there is an interesting contrast between \textit{noch nicht} \textit{alle} and \textit{noch nicht alle}, but it would need to digress too much to have a closer look at it.


\textsuperscript{11}See Jacobs 1982, p. 149.

\textsuperscript{12}See Jacobs 1982, p. 193.
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\[
\begin{align*}
&\left[ \forall zR(z, t_r) \land \exists t, \forall t < t_r \leftrightarrow \forall zR(z, t) \right] \\
&\left[ \exists t, \forall t < t_r \leftrightarrow \forall zR(z, t) \right] \\
&\left[ \exists zR(z, t_r) \land \exists t, \forall t < t_r \leftrightarrow \exists zR(z, t) \right] \\
&\left[ \exists t, \forall t < t_r \leftrightarrow \exists zR(z, t) \right]
\end{align*}
\]
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<td>??nicht alle schon</td>
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<tr>
<td>??niemand noch</td>
<td>(nicht alle mehr)</td>
<td>???{noch} nicht alle</td>
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\[
\begin{align*}
\neg \exists z R(z, t_r) &\quad \exists t_r \forall t (t < t_r \leftrightarrow \neg \exists z R(z, t)) \\
\neg \forall z R(z, t_r) &\quad \exists t_r \forall t (t < t_r \leftrightarrow \neg \exists z R(z, t)) \\
\neg \exists z R(z, t_r) &\quad \exists t_r \forall t (t < t_r \leftrightarrow \exists z R(z, t)) \\
\neg \forall z R(z, t_r) &\quad \exists t_r \forall t (t < t_r \leftrightarrow \exists z R(z, t)) \\
\end{align*}
\]

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<tr>
<td>$\exists t_r \neg R(z, t)$</td>
<td>$\exists \forall z R(z, t)$</td>
<td>$\exists \forall z \neg R(z, t)$</td>
<td>$\exists \forall z R(z, t)$</td>
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</table>
4 The problem

There is an important feature of the intuitive meaning of the phase structures illustrated in the third row of the table above which is not captured by the formalizations in the second row. Take, for instance, *einige schon* (some already) in equivalence class 3. The presupposition (44) of the formalization predicts correctly that during the presupposed negative phase (that means at all time points \( t \) which are not after \( t_* \)), we have \( \neg \exists z R(z, t) \).

In terms of our example, no people are present at \( t \) for all \( t \leq t_* \):

\[
\exists t_* \forall t (t_* < t \leftrightarrow \exists z R(z, t))
\]

As soon as we enter the positive phase and \( t \) is after \( t_* \), the existentially quantified formula \( \exists z R(z, t) \) becomes true. The problem is that the existential quantification can be made true by entirely different persons during the positive phase. Thus, the formalization allows for phase structures like (45) or (46). In both of them, the existential quantification is true during the positive phase after \( t_* \), even though people are arriving and leaving again. The number of people present may even decrease, as shown in (46).

(45) \[
\begin{array}{c|c|c}
\neg \exists z & \exists z & \exists z \\
\hline
\text{BEG} & t_* & \text{t} \\
\end{array}
\]

(46) \[
\begin{array}{c|c|c|c}
\neg \exists z & \exists z & \exists z & \exists z \\
\hline
\text{BEG} & t_* & \text{t} & \text{END} \\
\end{array}
\]

(47) \[
\begin{array}{c|c|c|c}
\neg \exists z & \exists z & \exists z & \exists z \\
\hline
\text{BEG} & t_* & \text{t} & \text{END} \\
\end{array}
\]

But clearly, what we intuitively mean by *einige schon* in sentences like (18) is the phase structure (47). In this phase structure nobody leaves once they arrive, and thus the number of persons present increases constantly with every person who arrives. Granted that a sentence such as (18) is typically not intended to explicitly exclude the possibility of someone leaving a (possibly big) party. But nevertheless there is a strong intuition that the expected continuous increase is due to the presupposition that nobody will leave (during a contextually relevant interval indicated by \text{BEG} and \text{END} in the above diagrams\(^{13}\)). We want the formal representation of

\(^{13}\)The contextually relevant interval indicated by \text{BEG} and \text{END} is not made explicit in the formalizations in the second row of the table on p. 13f (see p. 3).
(18) to account for this intuition and to rule out unwanted models such as (45) or (46).

Similar problems arise for each of the eight equivalence classes listed on p. 13f because in each of them, an existential proposition $\exists z R(z, t)$ or $\neg \forall z R(z, t)$ is required to be true either during the positive or during the negative phase. For instance, the formal representation of *nicht mehr alle* (no longer all) in equivalence class 6 allows for unwanted phase structures such as (48) as well as for correct phase structures such as (49).

![Diagram](attachment:image.png)

### 5 A solution

In order to rule out unwanted phase structures such as illustrated in the diagrams (45), (46) and (48) we have to take into account every horizontal line in these diagrams separately. That means, we have to take into account not only quantified formulae such as $\forall z R(z, t)$ or $\exists z R(z, t)$, but also non-quantified formulae such as $R(j, t), R(m, t)$, stating that John ($j$) is there, Mary ($m$) is there and so on.

There are several ways to describe and rule out the unwelcome features of the phase structures (45), (46) and (48). One way would be to require that for every person $z$, it must not be the case that the (contextually relevant) domain of times contains both the beginning of a maximal positive phase of $R(z, t)$ and the end of a maximal positive phase of $R(z, t)$. This condition precludes phase structures such as (45) and (46) in which for some $z$ there is a maximal positive phase of $R(z, t)$ such that both the beginning and the end of this maximal phase belong to the relevant domain of times. Moreover, this condition precludes phase structures such as (48) in which for some $z$ there are two distinct maximal positive phases of $R(z, t)$ such that the end of the first phase and the beginning of the second one belong to the relevant domain of times.

The unwanted phase structures could be precluded by adding the above condition as a further presupposition to the definition of the phase particles. Howe-
ver, the introduction of such an additional presupposition may appear to be an ad hoc solution which does not get to the heart of the problem. For instance, we would need to assume that the additional presupposition is also present when the phase particle is applied to sentences which do not contain any quantifiers. However, it seems to me that our problem is closely related to the fact that the phase particle is applied to a sentence whose subject is a quantifier. In what follows, I wish to propose one way of solving our problem without adding a new presupposition; instead, we will modify the given presupposition such that the solution of the problem is related to the fact that the phase particle is applied to a sentence whose subject is a quantifier.

Consider again the phase structure (47) denoted by *einige schon* (some already) in equivalence class 3. The formal representation (44) of this phase structure requires that the quantified formula $\exists z R(z, t)$ meet the presupposition of *already*, that is, that there be exactly one preceding negative phase during which $\exists z R(z, t)$ is false and exactly one ensuing positive phase during which this formula is true. Now, the unwanted phase structures (45) and (46) can be ruled out by requiring that not only $\exists z R(z, t)$ meet the presupposition of *already* but also the non-quantified formula $R(z, t)$ for every $z$. That is, for every person there must be exactly one negative phase during which she is absent and exactly one ensuing positive phase during which she is present. The duration of the negative phase may differ from person to person; for every $z$ there may be a different final point $t^*$ of the negative phase. Formally, this amounts to:

\[
\forall z \exists t^* \forall t (t^* < t \leftrightarrow R(z, t))
\]

This condition precludes the unwanted structures (45) and (46) in which there are for some persons two negative phases interrupted by a positive one. However, the condition in (50) is too strong, as the well-behaved phase structure (51) ($=\phi \phi $) does not meet it either. The reason is that in (51) not all individuals meet the presupposition of *already*, but only the four middle individuals $b, c, d$ and $e$. The two outer individuals $a$ and $f$ fail to meet the presupposition of *already* because they do not possess a positive phase.

Thus, we have to exclude the two irrelevant outer individuals, only requiring that the remaining relevant individuals meet the presupposition of *already*. In (51), the irrelevant individuals are those which are in a negative phase during all the time. Hence the remaining relevant individuals
\( b, c, d \text{ and } e \) can be picked out by the following formula:

\[
\exists t R(z, t)
\]

The same strategy works for the equivalence classes 4, 5, and 7. In all these equivalence classes, the irrelevant individuals do not possess a positive phase so that the relevant ones can be picked out by (52). Given this description of relevant individuals, we may require that not only the quantified formula \( \exists z R(z, t) \) meet the presupposition of the phase particle, but also that every relevant individual meet this presupposition.

In the remaining equivalence classes 1, 2, 6 and 8, the irrelevant individuals are those which do not possess a negative phase, so that the relevant individuals can be picked out by the following formula:

\[
\exists t \neg R(z, t)
\]

Apart from this difference, the strategy introduced for the equivalence classes 3, 4, 5 and 7 can also be applied to the equivalence classes 1, 2, 6 and 8. In the case of \textit{nicht mehr alle} in equivalence class 6, for instance, we have to require that not only the quantified formula \( \forall z R(z, t) \) meet the presupposition of \textit{still}, but also the relevant individuals picked out by (53).

For all eight equivalence classes, the irrelevant individuals can be characterized as those which do not undergo any change of polarity. Hence, the relevant individuals are those which possess a positive phase and a negative phase. In all eight equivalence classes, the individuals \( z \) relevant with respect to the relation \( R(z, t) \) can be picked out by the following formula:

\[
\text{rel}(z, R) = \exists t R(z, t) \land \exists t \neg R(z, t)
\]

As an aside, we might note that our formalization of the eight equivalence classes (the second row of the table on p. 13f) does not require that there exist any irrelevant individuals, though the diagrams in the third row of the table on p. 13f always contain irrelevant individuals (the two outer ones).

After having characterized the relevant individuals in all equivalence classes by (54), we now proceed to extend the definition of the phase particles in such a way that the presupposed phase structure holds not only for quantified formulae such as \( \forall z R(z, t) \) or \( \exists z R(z, t) \), but also for non-quantified formulae such as \( R(z, t) \) for all relevant individuals \( z \). According to definition (9), the phase particles are applied to a one-place predicate \( \lambda t. P(t) \) of times. This reflects the idea that in their basic meaning, phase particles take wide scope over a whole time-relative proposition (that is,
a predicate of times) without focussing on the constituents of the proposition. This idea, I submit, has to be given up in view of the problems described above. In order for these problems to be solved, we have to take into account not only quantified predicates of times such as \( \lambda t. \forall z R(z, t) \) or \( \lambda t. \exists z R(z, t) \), in which the subject position \( z \) is bound by a quantifier, but also several non-quantified predicates of times \( \lambda t. R(z, t) \), in which the subject position \( z \) is occupied by free variables. This is impossible as long as the phase particles are applied to quantified predicates of times such as \( \lambda t. \forall z R(z, t) \), as there is no way of removing the quantifier so as to obtain non-quantified predicates.

Instead, the phase particles should be applied separately to a two-place relation \( \lambda z \lambda t. R(z, t) \) obtaining between individuals and times and to a subject (external argument) occupying the left \( z \)-argument of this relation. This subject can be specified either as one of the quantifiers \( \forall x \), \( \exists x \) or as a relevant individual. That is, the one-place time-relative proposition \( \lambda t. P(t) \) is split up into a subject and a two-place relation \( \lambda z \lambda t. R(z, t) \).

The phase particle may be seen as providing the connecting link between the two parts of the time-relative proposition.

According to this view, phase particles do not take wide scope over a whole time-relative proposition in the natural language sentences considered in this paper. Rather, they resemble the behavior of a focus particle in that there is an interaction with a certain constituent of a proposition, that is, with the subject constituent realized by quantifiers such as \textit{alle} and \textit{einige}. Hence we shall assume that phase particles are applied separately to a subject and to a two-place relation of type \( \langle e, \langle i, t \rangle \rangle \) obtaining between individuals and times.

In order to make this strategy formally work, the quantifiers \( \forall x \) and \( \exists x \) should be treated as entities of the same logical type as individuals, since both individuals and quantifiers should occupy the subject position of \( \lambda z \lambda t. R(z, t) \) in the same way. This can be done by treating quantifiers and individuals as generalized quantifiers of type \( \langle \langle i, t \rangle \rangle, \langle i, t \rangle \rangle \). Such generalized quantifiers are applied to a two-place relation \( \lambda z \lambda t. R(z, t) \) of type \( \langle e, \langle i, t \rangle \rangle \), and yield a one-place predicate of times by binding the \( z \)-argument of type \( \langle e \rangle \) in this relation. The definitions of the generalized quantifiers \( \uparrow \forall \uparrow \), \( \uparrow \exists \uparrow \) and \( \uparrow z \uparrow \) are straightforward:

\begin{align}
(55) \quad & (a) \quad \uparrow \forall \uparrow =_{df} \lambda R. \lambda \lambda t. \forall z R(z, t) \\
& (b) \quad \uparrow \exists \uparrow =_{df} \lambda R. \lambda \lambda t. \exists z R(z, t) \\
& (c) \quad \uparrow z \uparrow =_{df} \lambda R. \lambda \lambda t. R(z, t)
\end{align}

For example, the assertion \( \forall z R(z, t_r) \) of \textit{alle schon} in the first equivalence class reads \( \langle \uparrow \forall \uparrow (R) \rangle \langle t_r \rangle \) in terms of generalized quantifiers. Taking \( \Omega \) and \( \Psi \) as variables for generalized quantifiers of the type defined in \((55)\), we can introduce a relation \( \Psi \preceq^K \Omega \) between generalized quantifiers which states
that $\mathfrak{P}$ is either identical\textsuperscript{14} to $\Omega$ or that $\mathfrak{P}$ is (identical to a generalized quantifier corresponding to) an individual relevant with respect to the relation $R$ in the sense of (54):

\begin{equation}
\mathfrak{P} \preceq^R \Omega \quad =_{df} \quad \Omega = \mathfrak{P} \lor \exists z (\text{REL}(z, R) \land \mathfrak{P} = \uparrow z \uparrow)
\end{equation}

Now, the presupposition of ALREADY and NOT-YET, formulated in (5), can be redefined in the following way:

\begin{equation}
\forall \mathfrak{P} [\mathfrak{P} \preceq^R \Omega \quad \supset \exists t, \forall (t_* < t \leftrightarrow (\mathfrak{P}(R))(t))]
\end{equation}

or shortly:

\begin{equation}
\forall \mathfrak{P} \preceq^R \Omega \quad \exists t, \forall (t_* < t \leftrightarrow (\mathfrak{P}(R))(t))
\end{equation}

This formula states that for all generalized quantifiers $\mathfrak{P}$ which are either identical to $\Omega$ or which are an individual relevant with respect to $R$, the formula $(\mathfrak{P}(R))(t)$ meets the presupposition of ALREADY. In the case of alle schon in the first equivalence class, for instance, $\Omega$ is specified as $\uparrow \forall \uparrow$ and (58) implies, first, that the formula $(\uparrow \forall \uparrow (R))(t)$ (i.e. $\forall z R(z, t)$) meets the presupposition of ALREADY; and, second, that the formula $(\uparrow z \uparrow (R))(t)$ (i.e. $R(z, t)$) meets this presupposition for every individual $z$ which is relevant with respect to $R$ in the sense of (54).

By analogy, the presupposition of STILL and NO-LONGER is obtained from (58) by inserting a negation immediately before the formula $(\mathfrak{P}(R))(t)$. In the case of einige noch in the fourth equivalence class, this presupposition states, first, that the formula $(\uparrow \exists \uparrow (R))(t)$ (i.e. $\exists z R(z, t)$) meets the presupposition of STILL and, second, that the formula $(\uparrow z \uparrow (R))(t)$ (i.e. $R(z, t)$) meets this presupposition for every relevant individual $z$.

Thus, the presupposition in (58) and its counterpart for STILL do what we want them to do, and the four phase particles can be redefined in terms of generalized quantifiers as follows:

\begin{equation}
\text{ALREADY} =_{df} \lambda R \lambda \Omega \lambda t_r . \left[ \forall \mathfrak{P} \preceq^R \Omega \quad \exists t, \forall (t_* < t \leftrightarrow (\mathfrak{P}(R))(t)) \right]
\end{equation}

\begin{equation}
\text{ALREADY} =_{df} \lambda R \lambda \Omega \lambda t_r . \left[ \forall \mathfrak{P} \preceq^R \Omega \quad \exists t, \forall (t_* < t \leftrightarrow (\mathfrak{P}(R))(t)) \right]
\end{equation}

\textsuperscript{14}Identity is to be taken here as the extensional identity of set theory.
I wish to conclude this paper by showing that the revised definition (59) does not only imply the standard wide scope definition of the phase particles in (9), but that it is even equivalent to this simpler definition under certain circumstances. More precisely, if the external argument \( \Omega \) is not a quantifier such as \textit{alle} or \textit{eineige} (\( \uparrow \forall \uparrow \) or \( \uparrow \exists \uparrow \)) but a single individual such as \textit{John} (\( \uparrow j \uparrow \)), the revised definition of the phase particles can be slightly modified such that it is equivalent to the standard wide scope definition. To this end, we have to take into account the domain of the generalized quantifier \( \Omega \), and to ensure that the relation \( \Psi \preceq^R \Omega \) introduced in (56) holds only if \( \Psi \) is \( \Omega \) or if \( \Psi \) is a relevant individual which belongs to the domain of \( \Omega \).

Intuitively, the domain of \( \uparrow \forall \uparrow \) and \( \uparrow \exists \uparrow \) is the whole domain of individuals of type \( \langle e \rangle \), the domain of \( \uparrow \text{all men} \uparrow \) and \( \uparrow \text{some men} \uparrow \) is the subset of \textit{men} in the domain of type \( \langle e \rangle \), and, crucially, the domain of \( \uparrow j \uparrow \) is the singleton \( \{j\} \). Formally, the domain of a generalized quantifier \( \Omega \) is the smallest set \( \Omega \) lives on (for this notion see Barwise and Cooper 1981, p. 178f). A generalized quantifier \( \lambda R \lambda t. \Omega \) of type \( \langle \langle e, t \rangle \rangle \) lives on a predicate \( \lambda z. E \) of type \( \langle e, t \rangle \) iff for all times \( t \) and for all \( \lambda z \lambda t. R \), it makes no difference whether \( \Omega \) is applied to the relation \( \lambda z \lambda t. R \) or to the relation obtained from \( \lambda z \lambda t. R \) by restricting the \( z \)-argument to the predicate \( E \):

\[
(60) \quad \Omega \text{ live-on } E = \text{df} \quad \forall \forall t R([\lambda z (\lambda z \lambda t. R(z, t)](t) \leftrightarrow [\Omega (\lambda z \lambda t. R(z, t) \land E(z))](t))
\]

15 Generalized quantifiers can be seen as NP-denotations resulting from the application of a determiner to an N-denotation, the N-denotation being a set of individuals (Barwise and Cooper 1981). For example, the determiners \textit{all} and \textit{some} are \( \lambda E \lambda R \lambda z ((E(z) \supset R(z, t))) \) and \( \lambda E \lambda R \lambda z ((E(z) \land R(z, t))) \) respectively. Then \( \uparrow \forall \uparrow \) is \( \lambda z (\lambda z z = z) \), \( \uparrow \exists \uparrow \) is \( \lambda z (\lambda z z = z) \), \( \uparrow \text{some men} \uparrow \) is \( \lambda z (\lambda z \text{man}(z)) \), and \( \uparrow j \uparrow \) is \( \lambda z (\lambda z j = z) \). Given this analysis of generalized quantifiers, the domain of \( \uparrow \forall \uparrow \), \( \uparrow \exists \uparrow \) and \( \uparrow j \uparrow \) is exactly the N-denotation \( \lambda z (z = z) \) or \( \lambda z (z = j) \) to which the determiner is applied in order to obtain the generalized quantifier in question.
The domain \([\Omega]\) of a generalized quantifier \(\Omega\) is the smallest set \(\Omega\) lives on. That is, \([\Omega]\) is an improper subset of all predicates on which \(\Omega\) lives. Or else, \([\Omega](z)\) is true iff \(z\) belongs to all sets on which \(\Omega\) lives:

\[
[\Omega](z) =_{df} \forall E((\Omega \text{ live-on } E) \supset E(z))
\]

It is not hard to verify that the following holds:\(^{16}\)

\[
(62) \quad \begin{align*}
&(a) \quad [\forall \exists](z) \iff [\exists \forall](z) \iff z = z \\
&(b) \quad [\exists \forall](z) \iff z = j
\end{align*}
\]

Finally, the relation \(\Psi \preceq^R \Omega\) is redefined by adding the condition \([\Omega](z)\) such that individuals which do not belong to the domain of \(\Omega\) are disregarded:

\[
(63) \quad \Psi \preceq^R \Omega =_{df} \Omega = \Psi \lor \exists z(\text{rel}(z, R) \land [\Omega](z) \land \Psi = \uparrow z \uparrow)
\]

If \(\Omega\) is \(\uparrow \forall \uparrow\) or \(\exists \uparrow\), the addition of the condition \([\Omega](z)\) does not have any consequences, as \([\Omega](z)\) is equivalent to the trivial condition \(z = z\). However, if \(\Omega\) is the generalized quantifier \(\uparrow j \uparrow\) corresponding to a single individual \(j\), \([\Omega](z)\) is tantamount to \(z = j\). Consequently, the only quantifier \(\Psi\) such that \(\Psi \preceq^R \uparrow j \uparrow\) is \(\uparrow j \uparrow\) itself: by (63). \(\Psi \preceq^R \uparrow j \uparrow\) \(\uparrow j \uparrow\) means that \(\Psi\) is either identical to \(\uparrow j \uparrow\) or a relevant individual belonging to the domain of \(\uparrow j \uparrow\); but the domain of \(\uparrow j \uparrow\) is the singleton \(\{j\}\). Hence, the presupposition (64a) can equivalently be reduced to the standard presuppositions (64b) and (64c) in which the phase particle can be taken to apply to a one-place predicate \(\lambda t. R(j, t)\) of times:

\[
(64) \quad \begin{align*}
&(a) \quad \forall \Psi \preceq^R \uparrow \forall \uparrow \exists t_s \forall t (t_s < t \iff (\Psi(R))(t)) \\
&(b) \quad \exists t_s \forall t (t_s < t \iff (\uparrow j \uparrow(R))(t)) \\
&(c) \quad \exists t_s \forall t (t_s < t \iff R(j, t))
\end{align*}
\]

\(^{16}\)To verify (62a), we have to show that neither \(\uparrow \forall \uparrow\) nor \(\exists \uparrow\) lives on any \(\lambda z.E(z)\) such that \(\neg \forall z.E(z)\). To this end, assume that there is an \(x\) such that \(\neg \forall z.E(z)\). Take \(\lambda z. M. R\) to be the relation \(\lambda z. M. R = z \land t = t\). In this case, we have for every time \(t\): \(\forall \uparrow (\lambda z. M. R(z, t))(t)\), whereas \(\forall \uparrow (\lambda z. M. R(z, t) \land E(z))(t)\) fails to hold. In view of (60), this means that \(\forall \uparrow\) does not live on \(\lambda z. E(z)\). Moreover, take \(\lambda z. M. R\) to be \(\lambda z. M. R = x \land t = t\). In this case, we have for every time \(t\): \(\exists \uparrow (\lambda z. M. R(z, t))(t)\), whereas \(\exists \uparrow (\lambda z. M. R(z, t) \land E(z))(t)\) fails to hold. This means that \(\exists \uparrow\) does not live on \(\lambda z. E(z)\).

To verify (62b), we have to show, first, that \(\uparrow j \uparrow\) lives on \(\lambda z. z = j\), and, second, that \(\uparrow j \uparrow\) lives on no \(\lambda z. E(z)\) such that \(\neg E(j)\). The first claim is obvious. To show the second claim, we assume \(\neg E(j)\) and take \(\lambda z. M. R\) to be the relation \(\lambda z. M. R = j \land t = t\). In this case, we have for every time \(t\): \(\uparrow j \uparrow (\lambda z. M. R(z, t))(t)\), whereas \(\uparrow j \uparrow (\lambda z. M. R(z, t) \land E(z))(t)\) fails to hold. This means that \(\uparrow j \uparrow\) does not live on \(\lambda z. E(z)\).
Thus, the standard wide scope phase particles in (9) turn out to be a special instance of the revised phase particles in (59). If the subject of the natural language sentence is a single individual such as John, the phase particles in (59) reduce to the wide scope operators defined in (9). Only when the subject is a quantifier such as alle or einige, we need to distinguish the subject from the subjectless time-relative proposition, and to take into account the interaction between the phase particle and the subject quantifier. If the subject is not a quantifier, we can safely continue assuming that, in their basic meaning, phase particles take wide scope over a whole time-relative proposition, that is, over a predicate of times.

References