Goals of the course: This course aims to introduce students to some of the most philosophically significant results of mathematical logic, beyond those covered in Intermediate Logic I (which is a prerequisite for this course). We will primarily focus on stating and proving the results in question, but will also devote some time (about 1/4 of the course) reading some philosophy of logic and discussing the philosophical significance of the results we have proved. The course covers some basic topics from model theory (compactness and Löwenheim-Skolem theorems, non-standard models), proof theory (natural deduction and sequent calculus, normalization theorems and cut-elimination), and the theory of definitions (Craig interpolation lemma, Beth’s definability theorem). The course does not cover Gödel’s incompleteness theorems, which deserve a course of their own, nor does it cover associated topics in recursion theory.

Work of the course: (1) Exercises of a logical nature. There will be about 5 bi-weekly problem sets over the course of the quarter: 75%. (2) Either (2a) a mid-length paper (8-10 pages) on some topic in the philosophy of logic discussed in the course or (2b) a final examination (2 hours, not take-home, closed book) on the logical material covered in the course: 25%. Regular attendance is required, and participation in discussion during the philosophical moments of the course will be necessary in order that these portions of the course be useful.

Texts: There is no text for the logical portion of the course. Attendance in class will be essential for this material. Readings for the philosophical interludes will be made available on e-reserve.

Approximate syllabus: I have not included dates here because I am not precisely sure at this point how much time each topic will require. This will unfold as the course progresses.

Topic 1: Model theory: Compactness and Löwenheim-Skolem theorems, non-standard models.
Philosophical interlude: Skolem’s paradox

Topic 2: Proof theory: Natural deduction and sequent-calculus, cut-elimination and normalization theorems.
Philosophical interlude: proof-theoretical accounts of the meaning of the logical constants

Topic 3: Theory of definition: Craig interpolation lemma, Beth’s theorem on implicit and explicit definability (proofs using techniques from both model theory and proof theory)
Philosophical interlude: explanation and reducibility in logic

Topic 4 (time permitting): Second-order logic and the limitations of first-order logic
Philosophical interlude: what is logic?