BERTRAND RUSSELL. *Foundations of Logic 1903–05.* The Collected Papers of Bertrand Russell, Volume IV. Alasdair Urquhart, ed. London and New York: Routledge, 1994. Pp. lii + 743 ISBN 0-415-09406-2.

Review by MICHAEL KREMER*

For over a decade the members of the Russell Editorial Project at McMaster University have been producing beautifully edited volumes of Russell's published and unpublished papers. This is a great service to the philosophical community, especially those of us who lack the resources for extended visits to the archives which were previously the only means of access to the multitude of unpublished manuscripts. These are often extremely revealing of the development of Russell's thought. The present volume is a case in point; the papers published here chart Russell's thinking over the three years from the publication of the *Principles of Mathematics* (*P of M*), in which appeared his most famous contribution to logic, Russell's paradox, to that of 'On Denoting' (*OD*—16 in this volume), in which appeared his most famous contribution to philosophy, the theory of descriptions. Most of the thirty-four items and six hundred odd pages published here concern one or both of these topics, and of these the majority are previously unpublished. These works vary greatly in style, clarity and polish; some are just rough notes, others are detailed technical attempts to work up a theory of classes and denoting. Overall they bring to light and focus attention on several points which had previously gone largely unnoticed. First, they make clear how closely connected the issue of denoting was for Russell to the problem posed by Russell's paradox announced in *P of M*. Many of his key ideas in developing theories of denoting arise out of his attempts to solve the paradox. Russell says in his *Autobiography* that he began to see the way to a solution to the paradox when he discovered the theory of descriptions;[1] the papers collected here show clearly just why this should be so. In fact this should not be surprising since already in *P of M* the connection between the paradox and the theory of denoting is implicit in the idea that classes are what are denoted by such paradigmatic denoting phrases as 'all men'. A second and related point is that logical concerns (having to do with the solution to the paradox first and foremost) are often the driving force behind shifts in the theories of denoting that Russell develops. Third, over the course of this period Russell develops a number of theories of denoting, and this complicates a great deal the interpretation of the eventual outcome in 'On Denoting', particularly as concerns what

* Department of Philosophy, University of Notre Dame, Notre Dame, Indiana 46556, U. S. A. michael.j.kremer.2Ond.edu

[1] Bertrand Russell. *The Autobiography of Bertrand Russell, Vol. 1: 1872–1914.* London: George Allen and Unwin, 1967, p. 152.

Russell takes himself to be *rejecting* there. I will return briefly to this point below.

The volume begins with a lengthy and extremely useful introductory essay by the editor, Alasdair Urquhart. Urquhart organizes his sketch of the development of Russell's logical views around the twin themes of the solution to the paradoxes and the search for a theory of denoting, and shows clearly the interconnection of these two problems. The introduction concludes by laying out the organizing principles of the edition. Urquhart takes his primary duty to be 'to provide an accurate, readable text of Russell's own words, unencumbered by textual apparatus' (p. xliii). I have of course been unable to check the accuracy of the text of the unpublished manuscripts but on a quick check of some of the previously published works ('On Denoting', 'Meinong's Theory of Complexes and Assumptions'), I was unable to find any inaccuracies, and I am confident in the overall quality of the text presented here.[2] Urquhart adds that 'some brief commentary is provided in headnotes and annotations'. The latter explain Russell's allusions, identify material referred to, note mistakes in quotations given by Russell, and Russell's own later marginal comments on the manuscripts; separate 'textual notes' identify variant texts and alterations in Russell's manuscripts. Urquhart modestly describes the headnotes as recording the origin of each paper, the surviving versions and the history of publication; but in fact these headnotes often include valuable summaries of key logical and philosophical points as well. The general index is also very useful and all in all the editing of the volume is superb.

The papers themselves are organized into five groups, with three appendices. The first three sections ('Early Foundational Work', 'The Zig-Zag Theory' and 'The Theory of Denoting') include fifteen unpublished manuscripts and *OD*. It is primarily in these papers that we can trace the development of Russell's logical thought in this period. The fourth section, 'Philosophy of Logic and Mathematics', comprises Russell's main published writings in this period (with the exception of *OD*). Several of these essays are also of direct relevance to the problems of denoting and the paradoxes, especially the long three-part essay on Meinong (**17**). Although these essays have all been published before, it is very useful to have them collected here in one volume with the unpublished manuscripts. The fifth section, 'Philosophical Reviews', contains some important pieces, particularly two

---

[2] I did find one (and only one) apparent error in the introduction. On p. xxii concerning manuscript 1a Urquhart writes 'Russell postulates that the "quadratic" functions are exactly those equivalent to functions having the form $F(x)$ or $F'(x)$; two propositional functions are to determine the same class provided neither are quadratic (*12.6).' First, this should presumably read 'two *equivalent* propositional functions', but even so *12.6, p. 9, does not say this. Rather it says that if two propositional functions are not quadratic, then if they determine the same class, they are equivalent. It is *12.61 on the next page which gives the converse as well.

reviews of Poincaré's *Science and Hypothesis* (**32, 33**), and a review of a collection of essays by Meinong and some of his followers (**34**). Finally the appendices include Russell's notes on Frege's 'way out' from the paradox, some comments on definitions from a dictionary of philosophical terms, and the French original of one of the published essays.

I have indicated above that this volume makes a significant contribution to our understanding of the development of Russell's thought, especially through making available unpublished work which was previously accessible only through a visit to the archives. I will close this review by indicating one point for which the unpublished manuscripts have great relevance—the interpretation of the much discussed 'Gray's Elegy Argument' (*GEA*) of *OD*. There have been many attempts to understand this 'inextricable tangle' over the years but even quite recent interpretations (including my own!) have failed to take account of the unpublished manuscripts which contain much material relevant to this issue.[3] A consensus seems to have developed that in the *GEA* Russell is not attacking Frege's sense-reference theory (as was held in Blackburn and Code's classic study),[4] but rather some earlier theory of his own, in some way related to the theory of denoting in *P of M*; but agreement ends there and divergent views on what exactly this earlier theory *is*, and exactly *how* Russell criticizes it, determine different readings of the *GEA*. Here however most interpreters have relied on the published texts of *P of M* and *OD* to arrive at a reconstruction of the theory being attacked in the *GEA*. Yet in the unpublished manuscripts Russell in fact develops several complicated theories of denoting, which promise to shed light on exactly what his intentions are in the *GEA*. Moreover, in the manuscript 'On Fundamentals' whose composition immediately preceded that of *OD*, an extremely complicated theory of denoting is developed and criticized, and some of the key ideas of the theory of descriptions of *OD* make their first appearance. Moreover some parts of the *GEA* of *OD* are lifted *verbatim* from the arguments of 'On Fundamentals', suggesting very strongly that the target in *OD* is also the theory developed and rejected in the first part of 'On Fundamentals'. Two recent essays by Russell Wahl

---

[3] Recent interpretations which do not make use of the unpublished manuscripts include: Pawel Turnau, 'Russell's argument against Frege's sense-reference distinction', *Russell* 11 (1991), 52–66; Michael Pakaluk, 'The interpretation of Russell's "Gray's Elegy" argument', in Andrew Irvine (ed.), *Russell and Analytic Philosophy*. Toronto: University of Toronto Press, 1993, pp. 37–65; and Michael Kremer, 'The argument of "On Denoting" ', *Philosophical Review* 103 (1994), 249–297. Gideon Makin, in 'Making sense of "On Denoting" ', *Synthese* 102 (1995), 383–412, mentions 'On Fundamentals' briefly but makes little use of the complex theory developed there. Peter Hylton's discussion in *Russell, Idealism and the Emergence of Analytic Philosophy* (Oxford: Clarendon Press, 1990, pp. 249–254) makes some use of the unpublished material but not as extensive as the work of Wahl and Rodríguez-Consuegra noted below.

[4] Simon Blackburn and Alan Code, 'The power of Russell's criticism of Frege: "On Denoting", pp. 48–50', *Analysis* 38 (1978), 65–77.

and Francisco Rodríguez-Consuegra follow out this idea,[5] and their work does indeed help to illuminate the development of Russell's thought here, but in my view the final story is yet to be told. The interpretative issues are extremely difficult; the theory initially developed, and then criticized in 'On Fundamentals' is itself both extremely obscure and complex, relying both on the idea that denoting phrases stand for 'denoting complexes' which themselves 'have' both meaning and denotation (a 'four-entity model' unlike the 'three-entity model' of denoting phrase, denoting concept and denoted object to be found in *P of M*), and on the further idea that complexes may 'occur as entities' or 'occur as meanings' in propositions, depending on the context in which they appear. I cannot attempt to disentangle these complexities here; but with the publication of the present volume, the texts that pose these puzzles are now available to a much wider scholarly audience than before.

ANNA SIERPINSKA. *Understanding in Mathematics.* London and New York: The Falmer Press, 1994. Pp. xv + 189. ISBN 0-7507-0334-2.

Review by JAMES ROBERT BROWN*

The issue of understanding and its relation to proof has been much discussed lately. The use of computer experiments, for example, has led some to claim a 'greater understanding' while others stress how misleading such work can be. (See, for instance, recent issues of *The Mathematical Intelligencer* and *Notices of the American Mathematical Society.*) Unfortunately, none of the recent developments in computer graphics, *etc.* that have given rise to the current debate are discussed here. Anna Sierpinska's book could have been written a century ago when Poincaré was musing on his discovery of Fuchsian functions. But this in itself is not to criticize what *Understanding in Mathematics* says, only to lament what it omits.

Sierpinska's work on understanding in mathematics was motivated by her interest in mathematical education and her commendable concern with improving it. However, she is concerned not only with how children come to understand elementary notions that others already understand but equally with how advanced researchers come to grips with novel ideas of which they have but a tenuous grasp.

Among numerous claims, the author says there are four principal mental operations involved in understanding: (1) *Identification,* a kind of recog-

[5] Russell Wahl, 'Russell's theory of meaning and denotation and "On Denoting" ', *Journal of the History of Philosophy* 31 (1993), 71–94; Francisco Rodríguez-Consuegra, 'A new angle on Russell's "Inextricable Tangle" over meaning and denotation', *Russell* 12 (1992–3), 197–207.

* Department of Philosophy, University of Toronto, Toronto M5S 1A1 Canada. jrbrown@epas.utoronto.ca