version of this geometry, independently derived. Carl Friedrich Gauss (1777–1855), often called the "*princeps mathematicorum*" (prince of mathematicians), simultaneously developed this geometry, calling it "non-Euclidean," but he kept his results private, fearing that the learned public was unprepared for this development. This type of geometry is now called *hyperbolic geometry*.

In 1766, Johann Heinrich Lambert (1728–1777) showed that hyperbolic geometry could be envisioned as the geometry that is valid on a sphere of imaginary radius. But being imaginary, this reasoning did not convince Lambert of the consistency of hyperbolic geometry. Lambert also recognized that the great circles on a sphere behave like lines without parallels, since every two "great circles" (paths on the sphere dividing it into two equal hemispheres) meet in two points. This idea gave rise to another type of non-Euclidean geometry called *elliptic geometry*, which was investigated more systematically by Bernhard Riemann (1826–1866). Riemann studied geometries on curved surfaces, including spheres and ellipsoids, and on curved spaces more generally.

Riemann's work was instrumental in convincing mathematicians of the value of non-Euclidean geometries. However, the question of their legitimacy remained. One of the chief nineteenth-century accomplishments in this arena was to show that if there were a contradiction in hyperbolic geometry, then there would be a contradiction in Euclidean geometry as well. It follows that if Euclidean geometry is consistent, then there is neither a proof nor a disproof of the parallel postulate available from the rest of Euclid's postulates; it is *independent* of the other postulates. Thus, attempts to prove Euclid's fifth postulate from the others were in vain, assuming the consistency of Euclidean geometry.

The work of Eugenio Beltrami (1835–1900) was critical in establishing the relative consistency of hyperbolic geometry with Euclidean geometry. He showed that the lines on a surface in Euclidean space called the *pseudo-sphere* behaved like straight lines in hyperbolic geometry. He also showed that the geometry on this surface could be mapped to a disk in the Euclidean plane. This provided a way to visualize hyperbolic geometry in Euclidean space, whose legitimacy was not in question. From this, geometers came to believe that Beltrami's disk constituted a *model* of hyperbolic geometry, thereby providing a Euclidean interpretation of the terms of hyperbolic statements. Geometers concluded from this reinterpretability that the meanings of geometrical terms are irrelevant to the information they convey. Thus the truth of geometrical statements was no longer at issue, but simply their consistency relative to an already-accepted theory. This remains the canon of admissibility for new mathematical theories today.

SEE ALSO Continuum; Extension; Infinity; Mathematics, Philosophy of; Measurement.

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NONSENSE

Nonsense, in general, means simply "lacking sense." As a term of philosophical criticism, it is often associated with the work of the Austrian philosopher Ludwig WITTGENSTEIN (1889–1951). For Wittgenstein, both in his early work (principally the *Tractatus Logico-Philosophicus* (*TLP*) and his later work (principally the *Philosophical Investigations* (*PI*), philosophy as traditionally conceived is, for the most part, nonsense. Philosophical activity as Wittgenstein conceived it, by contrast, is a kind of therapy for the illusion involved in the claims of traditional philosophy to make sense where, in Wittgenstein's view, no sense is being made. In the *Investigations*, he reports that his aim is "to teach you to pass from a piece of disguised nonsense to something that is patent nonsense" (*PI* §464). Philosophical activity is an activity of clarification: clarification both of the many things that individuals could mean by their words and of the ways in which their attempts to use words depart from or conflate those things. In a sense, then, the philosopher's treatment of a question is "like the treatment of an illness" (*PI* §255).

Wittgenstein's *Tractatus* Propositions. In the *Tractatus*, Wittgenstein's propositions serve a double purpose as representative of both the disease and its cure. The activity of clarification there is carried out by means of propositions that Wittgenstein describes as themselves nonsensical. He writes: "my propositions serve as elucidations in the following way: he who understands me eventually recognizes them as nonsensical, when he has used them—as steps—to climb up beyond them"

(*TLP* §6.54). Commentators have interpreted that remark, even at a broad level, in vastly different ways.

A central issue that is at stake in that disagreement is the nature of nonsense and what it can or cannot help one to achieve. For many commentators (Gertrude Elizabeth Margaret ANSCOMBE [1919–2001], Peter Geach [1919–], and P. M. S. Hacker [1939–] foremost among them), the nonsense-sentences of the *Tractatus* achieve their elucidatory and clarificatory purposes not merely by revealing the emptiness of the forms of the words to which individuals are attracted in philosophy, but by bringing them to appreciate certain fundamental and inexpressible features of the logical structure of the world and of language, features that underwrite the very possibility of making sense but that cannot themselves be captured in propositions that do make sense. They do this by deliberately flouting the rules of logical syntax, rules that determine what it is to make sense, and combining words in ways that violate the logical structure of language (and are therefore nonsense) precisely to draw attention to that structure. In this way, they gesture at what in Wittgenstein's terminology can only be shown and cannot be said (*TLP* §4.1212). Thus, they are nonsense of a quite special kind, a kind that is philosophically illuminating in a unique way.

A New Wittgenstein? A series of articles by James Conant and Cora Diamond (Conant 1998 and 2002; Diamond 1995) focusing on the philosophical viability of that notion of nonsense and the plausibility of its attribution to Wittgenstein has led increasing numbers of commentators to reject that picture of Wittgenstein's aims. Conant and Diamond argue that such a view involves adherence to what Conant calls a substantial view of nonsense, according to which there are (at least) two logically distinct kinds of nonsense: mere nonsense, consisting of words to which no meaning has been given in that context, and substantial nonsense, consisting of words whose meanings clash, thereby creating a determinate but illogical whole. Putative examples of the latter often discussed in the literature (Diamond 1995) include "Caesar is a prime number" and "Chairman Mao is rare." Against this, Conant and Diamond insist the idea of substantial nonsense is absurd: from a logical point of view, mere nonsense is the only kind of nonsense there is. The latter claim connects deeply with issues in philosophy of language concerning the relative priority of word-meaning and context in fixing the content of an utterance (Dain 2008) and has led to an extensive literature on the nature and communicative potential of nonsense (e.g., Kremer 2001; Moore and Sullivan 2003; Conant and Dain 2011).

Logical Positivism. Wittgenstein's attack on traditional philosophy in the *Tractatus* had a major influence on the philosophy of LOGICAL POSITIVISM and the verification theory of meaning, according to which, in broad terms, a sentence is meaningful if and only if it is either true by definition alone or is empirically verifiable (either in principle or in practice). However, repeated attempts (most notably by A. J. Ayer in his *Language, Truth, and Logic*) to articulate criteria of verifiability in such a way as to exclude as nonsense traditional philosophy, but not scientific generalities, met with failure, and the principle itself, being neither true by definition nor empirically verifiable, is widely held to imply its own nonsensicality. Moreover, and as with other putative criteria of meaningfulness, application of the verification principle always seems to come too late: to tell whether or not a given sentence is indeed verifiable one must first understand what the sentence says, but having done that it would be absurd then to say that the sentence in fact says nothing, and if one fails to understand it despite one's best efforts, then one does not have anything of the right category to which to apply the principle, and nothing more would be learned in applying it. Hence, the use for a theory of meaning imagined by the positivists, to identify criteria for distinguishing sense from nonsense, seems to be a chimera.

A final question connected to the use of nonsense as a term of philosophical criticism is whether it is plausible that one could ever suffer from illusions of meaning something in the manner suggested. Here, Wittgenstein credits Bertrand RUSSELL (1872–1970) with showing that the logical form of a sentence may differ utterly from its surface appearance and so with showing how, deceived by the surface appearance of a sentence, one may be led unwittingly into talking nonsense.

SEE ALSO EMPIRICISM, LOGICAL; VERIFIABILITY AND VERIFICATIONISM.

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NORMATIVITY

Normativity is the binding power of a precept, command, VALUE, or TRUTH. It answers the question of why this is obligatory, why this NORM must be followed, or why this truth obligates both credence and adherence.

Extrinsic Versus Intrinsic Normativity. Philosophers debate whether normativity is extrinsic or intrinsic. Why, for instance, should truth obligate credence, and why should credence obligate action in accord with what is known as true? Extrinsic normativity answers that credence and adherence are advantageous in some respect, for example, by pleasing an authority, by enabling one to acquire a reputation, by satisfying a desire, or even by enabling one to flourish in this life or the next. Intrinsic normativity answers that truth is an END—a GOOD in itself that cannot be acknowledged without also being known as obligatory.

The debate between intrinsic and extrinsic types of normativity lays bare a basic tension in human life: should one be good because one knows what is truly good or because it is advantageous in some respect? Should one live by reason or by DESIRE? This perennial question reaches back to the early days of ancient GREEK PHILOSOPHY when the term *philosopher* was coined, most likely by PYTHAGORAS (c. 570–c. 490 BC), to describe those focused on seeking the truth and living in accord with it.

The normativity of truth is inseparable from the question of moral normativity, which can likewise be either extrinsic or intrinsic. Consider, for instance, the norm to respect or LOVE one's neighbor as oneself. Its normativity would be extrinsic, if obligatory due to another's command or to some advantage. Its normativity would be intrinsic, if neighbors are worthy of being respected and loved, as argued by the GOLDEN RULE and Thomistic NATURAL LAW, or if respect could not be refused without violating the universality characteristic of the WILL's rational LIBERTY, as argued by Immanuel KANT (1724–1804).

Thomists and Kantians differ on whether intrinsic normativity is due to the good being prescribed or to the freedom with which a good is being prescribed. The reason for their difference on the question of normativity is their difference on the notion of freedom. Is freedom maximized when one is able to act in accord with what reason commands, as posited by Thomism? If so, then freedom is rational, and an objective ground exists for ETHICS. Or, is freedom maximized by being indifferent to the objects of CHOICE, as posited by Kantianism? If so, freedom is autonomous, and the ground of ethics is the will's self-legislation.

The debate between rationalist and voluntarist forms of intrinsic normativity is crucial for understanding the proper foundation of intrinsic normativity. This debate was formulated with special clarity in the seminal dialogues of PLATO (c. 428–c. 347), and especially in his highly influential *Republic*.

Rationalist Intrinsic Normativity. In the first book of his *Republic*, Plato sets the question of normativity in terms of JUSTICE. Are one's obligations to others determined by some objective good or by choice alone? Plato answers that without wisdom, without adherence to the truth, choice is ultimately impotent. It is the objective truth about the good—and not choice—that generates wisdom and normativity. Plato thereby identifies normativity with truth, and truth with the good: "the good is the principle of being and intelligibility." As a result, what is truly good is normative.

Aristotle (384–322 BC) furthers the identification of the good with normativity by arguing that the goodness of the divine unmoved mover initiates motion by being loved. This teleological aspect of the good identifies it as an end necessitating—or obligating—that the means of its achievement be undertaken. This teleological aspect of good is retained in Aristotelian ethics because the end of ethics is achieving the good of *eudaimonia* (HAPPINESS) understood "primarily as contemplating God and secondarily as acting morally." Aquinas concurred not only by arguing that the obligation to do and to pursue the good is self-evident (*Summa theologiae* 1, q. 94, a. 2c), but also by adding that loving God above all and neighbors as oneself are self-evident obligations known to reason either from nature or from the faith (*ST* 1–I2, q. 100, a. 3, ad 1).

Voluntarist Intrinsic Normativity. This characterization of normativity as intrinsically related to goodness was already under fire at the time of Aquinas's death by the voluntarist HENRY OF GHENT (c. 1217–1293).