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The Conceivability of Platonism[†]

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It is widely believed that platonists face a formidable problem: that of providing an intelligible account of mathematical knowledge. The problem is that we seem unable, if the platonist is right, to have the causal relationships with the objects of mathematics without which knowledge of these objects seems unintelligible. The standard platonist response to this challenge is either to deny that knowledge without causation is unintelligible, or to make room for causal interactions by softening the platonism at issue. In this essay I argue that the idea of causal relations with fully platonist objects is unproblematic.

The platonist claims that the objects of mathematics inhabit a necessary and unchanging realm of non-spatial eternal entities. If he is right, how can we know anything about them? Our best epistemological theory tells us that in order to come to have knowledge about something, we must causally interact with that thing; and causal interaction with abstract objects seems impossible. In this way the objects of mathematics would appear, if the platonist is correct, to be 'cut off from all links with the reflecting subject' [Bernays, 1983, p. 259]. In short, the platonist seems forced to choose between abandoning his abstract objects and abandoning his knowledge of those objects.

This is the problem of access, brought to the fore by Paul Benacerraf in his essay 'Mathematical truth' [Benacerraf, 1983]; in this brief essay I suggest a way of understanding Benacerraf's problem, and, given this understanding, a solution.

Many philosophers, of course, find platonism uncongenial quite apart from any epistemological worries; on their view, the theory should be rejected on purely metaphysical (typically, naturalist) grounds. I will set aside these and other worries about platonism, and focus exclusively on the problem of access—on what Jerrold Katz [1998, p. 25] called the charge of 'epistemological malfeasance' against platonism. Benacerraf, for his own part, takes the problem of access as the chief problem with

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what is, by his own lights, an otherwise highly attractive theory;[1] for those who agree with Benacerraf, removing that problem will presumably clear the road to an acceptance of platonism.

The claim that we cannot causally interact with abstract objects is subject to two interpretations, and the solution I propose depends on carefully distinguishing between them.

On the first interpretation, the challenge at issue is this: laws of conservation of energy which we know to be true would be violated if there were causal interactions between abstract objects and ourselves; therefore, there cannot be any such interactions. The 'cannot' here is either causal (the existence of these interactions would violate the laws of nature) or epistemic (the existence of these interactions is incompatible with what we know, empirically, to be the case). Call this *the empirical challenge* to platonism.

The force of the empirical challenge can be called into doubt. We can question whether we have good empirical reasons to believe that energy is conserved across the whole universe.[2] Moreover, even if energy is

[1] '[Anti-platonist accounts are] almost always motivated by epistemological concerns' [Benacerraf, 1983, p. 409]; 'The principal defect of [platonism] is that it appears to violate the requirement that our account of mathematical truth be susceptible to integration into our overall account of knowledge' [1983, p. 412].

[2] The speculative quality of conservation claims is captured in the following comment by the physicist Richard Feynman:

> Why are we so confident that, because we have checked the energy conservation here, when we get a new phenomenon we can say it has to satisfy the law of conservation of energy? ... the only utility of science is to go on and try to make guesses. So what we always do is to stick our necks out, and in the case of energy the most likely thing is that it is conserved in other places. [Feynman, 1965, p. 76]

John Foster, speaking of the assertion of causal closure in the context of the mind-body problem, makes the following trenchant remarks:

> [The case for closure] is a conspicuously weak one ... [The evidence for it] has to be evaluated in the light of what we independently know, or have reason to believe, about the special relationship between the human body and the human mind ... [I]f all we knew about the brain was that it was a physical object of a certain physical type, it would be reasonable, prior to direct investigation, to suppose that its functioning would turn out to be explicable in terms of the same laws of physics and chemistry as other physical things ... but, in fact, we already know that the brain is the seat of the mind; and all that we ... know about ourselves suggests that the mind, qua mind, has a causal influence on behavior ... While there has been considerable research on the brain, it could hardly be thought sufficiently extensive and revealing to provide, on its own, a strong case against [the existence of violations of conservation of energy]. [Foster, 1991, pp. 200, 199]

conserved it is far from obvious how this is supposed to be incompatible with causal relations with abstract objects.[3]

But, in any event, the empirical challenge is not Benacerraf's challenge. Benacerraf [1983, p. 403] tells us that it is *unintelligible* how we could have mathematical knowledge if the objects of that knowledge were abstract. Penelope Maddy, summarizing Benacerraf's worry [Maddy, 1990, p. 37], asks, 'how *can* entities that do not even inhabit the physical universe take part in any causal interaction whatsoever?' (emphasis added). And here is Michael Resnik's representation of Benacerraf: 'If mathematics is about abstract entities that exist outside of space and time, then it's an utter mystery as to how we can access them and acquire mathematical knowledge.' [Resnik, 2004, p. 32] Notice that Resnik does not say: we have empirical reason to think that the world does not in fact work that way. He says that it is an *utter mystery* how it *could* work that way. On this second, correct, intepretation of Benacerraf's claim, the challenge at issue is this: we cannot make any *sense* of the idea of causal interactions between abstract objects and ourselves. Such interactions are inconceivable, given the radically different natures of the relata. Call this *the a priori challenge* to platonism.

In pressing the *a priori* challenge, Benacerraf is articulating an ancient worry about platonism;[4] philosophers were troubled about mathematical knowledge for these reasons long before conservation laws were so much as imagined. The solution I will propose to the problem of access is directed at that problem *understood in terms of the a priori challenge*.

Benacerraf means, by causation, *efficient* causation—the relation of 'something making something else happen' [Searle, 2001, p. 40]. And he assumes—and I will join him in assuming—that our states of knowledge are, or causally depend on, states of our brain. So the allegation is that there is something inconceivable about abstract objects entering into efficient causal relations with the physical objects—our brains—whose states must underwrite our knowledge of those abstract objects. Benacerraf, to repeat, wonders: how could abstract objects cause

I think analogous points can be made in the context of the issue of mathematical knowledge. The naturalist will no doubt regard this gambit as a rearguard action, delaying the inevitable naturalist conclusion. But if his impatience is explained by his commitment to his naturalism—and I think it usually is—then he must, in the present context, be arguing in a circle. The conclusion of the epistemological argument is that platonism is false; so the assumption that platonism is false had better not show up in the premises. For more on the pernicious effects of naturalist commitments on discussions of the epistemological problem, see [Katz, 1998, pp. 25–28].

[3] The idea that efficient causation is, or essentially involves, energy transfer is defended by some (*e.g.*, Quine; see footnote 5, below); but it is hardly the dominant view.

[4] Mark Steiner notes the ancient pedigree of the problem in his [1973] and [1992].

anything to happen or obtain in the material world? How *could* the world be that way? It is a form of question familiar in philosophy and the sciences, expressing a sense of bafflement, a kind of metaphysical vertigo. The idea is that there is something absurd or uncanny or mysterious as such about the idea of causal relations between abstract objects and ourselves.

But why, exactly, cannot abstract objects cause us to know things about them? It is, of course, highly plausible to think that objects have causal powers only 'under a description'—that 'it is a constraint on the notion of [efficient] causation that wherever some object x is cited as a cause, there must be some feature or property of x or some event involving x that functions causally.' [Searle, 2001, p. 82] So the question is: why (for example) cannot the fact that the number 17 is prime cause us to (*i.e.*, explain, in terms of efficient causation, why it is that we) believe that the number 17 is prime, if that number is an abstract object?

We might think that the grounds of the challenge are to be found in the idea that efficient causation must proceed by *contact* (or *impact*)—that contact necessarily involves touch, and that non-spatial objects cannot touch anything. But why think that efficient causation must proceed by contact in this (touch-entailing) sense? Efficient causal relations unsupported by contiguity relations are perfectly intelligible and (therefore) apparently possible, even if they are not actual. Fred Dretske makes this point (about causation generally, and causal theories of knowledge and perception in particular) in this way:

> As a child I never found the visual exploits of Superman (seeing through buildings [without any transmission of light between the objects and the subject]) incoherent or logically paradoxical . . . [T]his was just a fanciful ('fanciful' because, as things stand, no one can see things in that way) account of an extraordinary individual who could see things in ways that no one else could. [2000, p. 111]

If 'action at a distance'—that is, one physical object causally affecting another without (even indirectly) touching it—is intelligible, then the problem with the idea of causal interactions between ourselves and abstract objects is not to be found in the absence of impacts in the mathematical case (since these would be absent in cases of action at a distance too), but must be sought elsewhere.

It might be thought that the strong—*i.e.*, metaphysical—impossibility of causal interactions between abstract objects and ourselves lies not in the notion of touch, but in another element in the idea of efficient causation: the idea of force or energy. Roughly, the worry is that efficient causation essentially involves energy transfer, so that in order to make

sense of efficient causal relations between the objects of mathematics and ourselves, we cannot accept the platonist claim that they are unchanging.

But even if it is granted that energy transfer is strongly necessary for efficient causation, we still do not have the materials of an argument for the strong impossibility of causal relations with abstract objects. Here is why. It may, perhaps, be strongly impossible for abstract objects to *receive* energy; for (as we noted at the beginning of our discussion) it is a principle of classical platonism that the objects of mathematics are unchanging, and it may be strongly necessary that any object receiving energy must change. But we do not have any reason to think that it is strongly necessary that if abstract objects impart energy to us (as part of the process of acquiring knowledge about them) they must themselves receive energy. We *would* have such a reason if Newton's third law of motion (in Newton's words, 'If a body impinge upon another, and by its force change the motion of the other, that body also (because of the equality of the mutual pressure) will undergo an equal change, in its own motion, toward the contrary part'), or anything like it, were a strongly necessary truth. But it is not; it is just an empirical truth. There is no contradiction, or any other conceptual or metaphysical difficulty, in accepting the claim that abstract objects impart energy to us, and thereby change us, without themselves receiving any energy or suffering any change. The idea of an 'unmoved mover' is perfectly intelligible; in fact, this lack of reciprocity is part of the very idea of classical platonism. Thus Frege the platonist remarks:

> The hammer ... is gripped, it undergoes pressure, and thus its density, the disposition of its parts, is locally changed. There is nothing of all this with [abstract entities] ... [T]here is lacking here something that we observe everywhere in physical process—reciprocal action. [Frege, 1997, pp. 344–345]

The platonist's proposal is that we grasp or perceive the number five, and thereby come to know that it is prime, but though in grasping it we are changed, we do not cause it to change. Such an asymmetric relation may be empirically impossible, but it is certainly not strongly impossible.[5]

[5] There is a second, closely related anti-platonist argument:

1. For something to impart energy, it must have energy to impart; but
2. If Einstein is right, energy has a mass equivalent;
3. Abstract objects cannot have mass; therefore
4. It is strongly impossible for abstract objects for impart the energy necessary for efficient causation.

Like the other argument that invokes the notion of energy, this argument fails because one of its premises (in this case, (2)), is not strongly necessary, but merely empirical.

Finally, it might be thought that, because causation happens at a time, the thing doing the causing cannot be atemporal, as the platonist takes mathematical objects to be. For example, the platonist Jerrold Katz tells us that 'it is ... senseless to suppose that we can be acquainted with atemporal abstract objects ... [A]cquaintance requires a point of contact, some temporal position that both we and the object occupy, but there cannot be such a point in the case of objects that have no temporal location ... ' [Katz, 1998, p. 15] Here we need to distinguish two senses of 'atemporal'. Think of the paradigmatic platonic theory: Plato's theory of the Forms. Plato thought (for example) that when a beautiful chalice is created, the Form of Beauty is instantiated. The creation of the chalice happens at a time—say, Thursday afternoon. It follows that for Plato something happens to the Form of Beauty on Thursday afternoon: to wit, it is instantiated. But this is hardly a retraction of his platonism. In this sense of 'temporal', platonists can admit that abstract objects are temporal; indeed, they will usually insist on it, since otherwise the phenomena we encounter in the spatio-temporal realm cannot be explained by appeal to those objects (as Plato, for one, thought they could). They can still be necessary, permanent and unchanging, and this is the sense of 'atemporal' operative in classical platonism.

To sum up: contiguity relations and reciprocal action are contingent features of efficient causation; so their strong impossibility in the case of abstract objects cannot support the claim that efficient causal relations with abstract objects are strongly impossible.[6]

For discussion of a Quinean argument that combines that the 'reciprocity' objection with the 'mass-energy' objection, see [Hart, 1979].

It is perhaps worth noting that doubts about the idea of an 'unmoved mover' are often voiced by platonists themselves. Plato, for example, tells us that 'if knowing is to be acting on something, it follows that what is known must be acted upon by it, and so, on this showing, reality when it is being known by the act of knowing must, in so far as it is known, be changed owing to being so acted upon—and that, we say, cannot happen to the changeless' (*Sophist*, 248; quoted in [Steiner, 1992]). But why did Plato think that 'knowing is to be acting on something'? It is highly plausible that knowing essentially involves *being acted upon by* the thing known (the 'logical' echo of this fact is the fact that knowing has 'mind to world' direction of fit), but I see no reason to think that it *essentially* involves *acting on* the things known.

[6] Here is another possible challenge to my central claim: causal relations require the truth of counterfactual claims that are, in the case of necessary abstract objects, false. For events in our brains to be caused by how things are with numbers, it would (according to this challenge) have to be true that if things had been different with numbers, the events in question would not have occurred—and things *cannot* be different with numbers if their properties are, as the platonist claims, necessary. The first point to make in reply to this challenge is that, whatever else we make of it, it is not part of Benacerraf's problem—so we can solve that problem (which is our primary aim here) without addressing the challenge. Second, there is plenty to be said against the challenge. I shall not go over this terrain in any detail here, but it should be noted that both premises of the challenge—the

Some will be tempted to say: if mathematical objects figure in the explanation of the motions of physical objects in the universe (*e.g.*, the neurons in human brains), then it follows trivially that they are themselves physical, and not abstract. The move is common among naturalists. John Searle, for example, says that

> For us [naturalists], if it should turn out that God exists, that would have to be a fact like any other. To the four basic forces of the universe—gravity, electromagnetism, weak and strong nuclear forces—we would add a fifth, the divine force ... [I]t would still be all physics, albeit divine physics. If the supernatural existed, it too would have to be natural. [Searle, 1998, p. 35]

This sort of terminological appropriation, whether it is applied to God, numbers, or anything else, fails to address the underlying question. By decreeing that the word 'natural' (or 'physical') is to be applied to any phenomenon we discover, the naturalist robs naturalism of any content relevant to the substantive dispute between naturalists and those who disagree with them. I have claimed that efficient causal relations between non-spatial, necessary, eternal, unchanging objects and spatial, contingent, changing objects are strongly possible, and I have used the word 'abstract' to refer to the former sort of objects, and 'physical' or 'material' or 'concrete' for the latter sort. But the truth of my claim is not affected, or illuminated, if we decide to use these words in some other way instead.

To review: Benacerraf is baffled by the idea of abstract objects acting on our brains to produce knowledge of those objects; the truth of this idea seems to him inconceivable. Benacerraf is not alone in thinking this: in fact, every platonist that I am aware of joins him in his bafflement. Many platonists have tried to hold onto their platonism by rejecting the idea that the objects of mathematics need to influence our brains causally in order to produce knowledge.[7] Others have granted that mathematical objects, in order to be known, need to operate causally on our brains, and have consequently offered a 'new, improved' abstract object: one that has spatio-temporal location after all.[8] All agree with Bob Hale when he

claim that causal relations need to support counterfactuals, and the claim that we cannot make sense of counterfactuals in the case of necessary domains—are controversial. See [Katz, 1998, pp. 36–39] for further discussion and references.

[7] In very different forms, this proposal can be found in [Steiner, 1973], [Quine, 1981], [Katz, 1998], [Tait, 1986], [Yourgrau, 2006], and [Sosa, 2002]. Skepticism about the causal theory of knowledge is hardly confined to platonists; in the words of the (anti-platonist) Hartry Field, 'almost no one believes [it] any more'. [1989, pp. 25–27]

[8] This sort of solution is most prominently found in the work of Penelope Maddy. See [Maddy, 1990].

says that 'the claim that we can and do *literally perceive sets* [or other abstract objects] ... surely is [preposterous], if sets are conceived, as they usually are, to be abstract objects, outside of time and space'.[9] I have argued that this claim is not preposterous, that it is in fact perfectly intelligible.

There is a certain historical irony in the situation. Benacerraf's challenge to platonism has been most popular with empirically minded philosophers, who see the challenge's invocation of the causal theory of knowledge as a way of bringing back to earth the rationalists (like Frege, Gödel, and, perhaps, Plato himself) who accept the metaphysical excesses of mathematical platonism. But the challenge is, in fact, itself a specimen of high rationalism, grounded in a principle inherited directly from philosophers like Descartes, Spinoza, and Leibniz; *viz.*, that 'entities of ... radically diverse types [cannot] really cause changes to happen in one another.'[10] Acceptance of this principle is what caused Descartes to struggle with, and Spinoza and Leibniz to reject, mind-body interactionism. But this principle, whether endorsed by rationalists or empiricists, is false. Conceptually speaking, there is, on reflection,

[9] [Hale, 1993, p. 46]; emphasis in original. If we build, into our very notion of perception, the idea of a *sensual presentation*—*e.g.*, visual experiences, smells, *etc.*—then (given the sensual experiences we in fact have) it will of course follow that we do not perceive abstract objects (though it will not follow that no thinking being *could* have such perceptions). But the point at issue remains unaffected: knowledge of abstract objects that involves efficient causal relations between those objects and ourselves is strongly possible.

[10] [Delahunty, 1985, p. 178]. Acceptance of this rationalist principle, and of the closely related principle that satisfying causal explanations must reveal what I have called *strong* necessity, is evident in the writings of Thomas Nagel and Colin McGinn, in their work on the mind-body problem. Nagel, for example, asserts that we have the impression of strong necessity in the case of causal relations within the physical universe, but that we lack it, and seem unable even to imagine it, when we turn to the relation between mind and body (see, *e.g.*, [Nagel, 1986, pp. 48–49]). I find Nagel's view of physical causation puzzling. We do not need to accept Hume's skepticism about causal relations in order to agree with his point that 'The contrary of every matter of fact is still possible ... *[T]hat the sun will not rise tomorrow* is no less intelligible a proposition, and implies no more contradiction, than the affirmation, *that it will rise*.' [Hume, 1977, p. 15] John Foster articulates the problem with Nagel's view of physical causation, and its implications for mind-body causation, in this way:

> In the physical realm too our explanation of causation has to terminate in the postulation of certain causal laws, without any further explanation ... of why these laws obtain. So why should the dualist be required to do more? Why should he be called on to offer a deeper mode of explanation than that which is available to physical science? ... Causation in the physical realm is not amenable to any distinctively deep explanation ... and, if this is so, then we have not yet identified anything about dualistic causation which renders it conceptually problematic. [Foster, 1991, pp. 161–162]

simply no problem at all with the idea of something being affected by an entity radically dissimilar from it. In particular, there is no problem with the idea of abstract objects effecting changes in us; there is no conceptual difficulty with the idea that they impart energy to our brains, and that they do this 'at a distance', *i.e.*, without the benefit of contiguity relations. In a way, we had the answer all along. Causality is an empirical relation, and Benacerraf's claims are claims about what causal relations obtain—but the argument, or rather the sense of perplexity, behind those claims is not empirical, but purely *a priori*. It is no wonder, then, that the *a priori* problem turns out to be a kind of illusion.

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Similar remarks are made by Searle:

> In the end when we investigate nature we find: this is what happens. This is how it works. If you want to know how/why a body falls, the standard answer is to appeal to gravity. But if you want to know how/why gravity works, I am told that the question is still not answered. But suppose it were, suppose we had a unified field of everything that explained gravity, electromagnetism, and everything else. That would still leave us with the question, why are the data accounted for by this theory and not some other? In the end, how/why questions stop with theories that state how nature works and the mechanisms according to which it works. [Searle, 2005]

The point is this: relations between bodies and bodies, *contra* Nagel, are not strongly necessary; so there is no reason, grounded in a contrast with physical causation, to demand strongly necessary relations between minds and bodies, or—to apply the point to our present discussion—between abstract mathematical objects and brains.

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